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MUSLIM CONTRIBUTIONS TO MATHEMATICS

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When we talk about Muslim contributions to mathematics we are usually referring to the years between 622 and 1600 ce. This was the golden era of Islam when it was influential both as a culture and religion, and was widespread from Anatolia to North Africa, from Spain to India.

Mathematics or "the queen of the sciences" as Carl Friedrich Gauss called it, plays an important role in our lives. A world without mathematics is unimaginable. Throughout history, many scholars have made important contributions to this science, among them a great number of Muslims. It is beyond the scope of a short article like this one to mention all the contributions of Muslim scholars to mathematics; therefore, I will concentrate on only four aspects: translations of earlier works, and contributions to algebra, geometry, and trigonometry. In order to understand fully how great were the works of scholars in the past, one needs to look at them with the eye of a person of the same era, since things that are well-known facts today might not have been known at all in the past.

There has never been a conflict between science and Islam. Muslims understand everything in the universe as a letter from God Almighty inviting us to study it to have knowledge of Him. In fact, the first verse of the Qur'an to be revealed was:

Read! In the Name of your Lord, Who created... (Alaq 96:1).

Besides commanding us to read the Qur'an, by mentioning the creation the verse also draws our attention to the universe. There are many verses which ask Muslims to think, to know, to learn and so on. Moreover, there are various sayings of the Prophet Muhammad, peace be upon him, encouraging Muslims to seek knowledge. One hadith says, "A believer never stops seeking knowledge until they enter Paradise" (al-Tirmidhi).

In another hadith, the Prophet said, "Seeking knowledge is a duty on every Muslim" (Bukhari). Hence it is no surprise to see early Muslim scholars who were dealing with different sciences.

Translations

Prophet Muhammed (pbuh) said, "Knowledge is the lost property of a Muslim; whoever finds it must take it" [1]; hence Muslims started seeking knowledge. One way they did this was to start translating all kinds of knowledge that they thought to be useful. There were two main sources from which Muslim scholars made translations in order to develop the field of science, the Hindus and the Greeks. The Abbasid caliph al-Mamun (804–832) had a university built and ordered its scholars to translate into Arabic many works of Greek scholarship. Between 771 and 773 CE the Hindu numerals were introduced into the Muslim world as a result of the translation of Sithanta from Sanskrit into Arabic by

Abu Abdullah Muhammad Ibrahim al-Fazari. Another great mathematician, Thabit ibn Qurra, not only translated works written by Euclid, Archimedes, Apollonius, Ptolemy and Eutocius, but he also founded a school of translation and supervised many other translations of books from Greek into Arabic. While Hajjaj bin Yusuf translated Euclid's *Elements* into Arabic, al-Jayyani wrote an important commentary on it which appears in the *Fihrist* (Index), a work compiled by the bookseller Ibn an-Nadim in 988. A simplified version of Ptolemy's *Almagest* appears in Abul-Wafa's book of *Tahir al-Majesty* and *Kitab al-Kamil*. Abu'l Wafa Al-Buzjani commented on and simplified the works of Euclid, Ptolemy and Diophantus. The sons of Musa bin Shakir also organized translations of Greek works.

These translations played an important role in the development of mathematics in the Muslim world. Moreover, the ancient Greek texts have survived thanks to these translations.

Algebra and geometry

The word "algebra" comes from "Al-Jabr", which is taken from the title of the book *Hisab Al-Jabr wal Muqabala* by Muhammad ibn Musa al-Khwarizmi (780–850). Al-Khwarizmi, after whom the "algorithm" is named, was one of the great mathematicians of all times. Europe was first introduced to algebra as a result of the translation of Khwarizmi's book into Latin by Robert Chester in 1143. The book has three parts. The first part deals with six different types of equations:

$$(ax^2 = bx) ; (ax^2 = b) ; (ax = b) ; (ax^2 + bx = c) ; (ax^2 + c = bx) ; (bx + c = ax^2)$$

Khwarizmi gives both arithmetic and geometric methods to solve these six types of problems [2]. He also introduces algebraic multiplication and division. The second part of *Hisab Al-Jabr* deals with mensuration. Here he describes the rules of computing areas and volumes. Since Prophet Muhammad, peace be upon him, said, "Learn the laws of inheritance and teach them to people, for that is half of knowledge,"[3] the last and the largest part of this section concerns legacies, which requires a good understanding of the Islamic laws of inheritance. Khwarizmi develops Hindu numerals and introduces the concept of zero, or "*sifr*" in Arabic, to Europe. The word "zero" actually comes from Latin "*zephirum*," which is derived from the Arabic word "*sifr*."

The three sons of Musa bin Shakir (about 800–860) were perhaps the first Muslim mathematicians to study Greek works. They wrote a great book on geometry, *Kitab Marifat Masakhat Al-Ashkal* (The Book of the Measurement of Plane and Spherical Figures), which was later translated into Latin by Gerard of Cremona. In the book, although they used similar methods to those of Archimedes, they move a step further than the Greeks to consider volumes and areas as numbers, and hence they developed a new approach to mathematics. For example, they described the constant number pi as "the magnitude which, when multiplied by the diameter of a circle, yields the circumference." [4]

A well-known poet, philosopher and astronomer Omar Khayyam (1048–1122) was at the same time a great mathematician. His most famous book on algebra is *Treatise on the Demonstration of Problems of Algebra*. In his book besides giving both arithmetic and geometric solutions to second degree equations he also describes geometric solutions to third degree equations by the method of intersecting conic sections. He also discovered binomial expansion [26]. His work later helped develop both algebra and geometry.

Thabit bin Qurra (836–901) was an important mathematician who made many discoveries in his time. As mentioned in the *Dictionary of Scientific Biography* [5] he "played an important role in preparing the way for such important mathematical discoveries as the extension of the concept of number to (positive) real numbers, integral calculus, theorems in spherical trigonometry, analytic geometry, and non-Euclidean geometry. In astronomy Thabit was one of the first reformers of the Ptolemaic system, and in mechanics he was a founder of statics."

To give an idea of his importance, we will just give here, without details, one of his theorems on amicable numbers. Two natural numbers m and n are called "amicable" if each is equal to the sum of the proper divisors of the other:

for $n > 1$, let $p_n = 3 \cdot 2^{2n-1}$ and $q_n = 9 \cdot 2^{2n-1} - 1$. If p_{n-1} , p_n and q_n are prime numbers, then $a = 2n \cdot p_{n-1} \cdot p_n$ and $b = 2^n q_n$ are amicable. [6]

Abu Kamil (about 850–930), an Egyptian mathematician, wrote the *Book on Algebra* which consists of three parts:

- (1) Solutions of quadratic equations,
- (2) Application of algebra to geometry,
- (3) Diophantine equations. [7], [8]

He improved the work of Khwarizmi and applied algebraic methods to geometry. His research was on quadratic equations, multiplication and division of algebraic quantities. His work also includes addition and subtraction of radicals. He found the following formulas:

$$ax \cdot bx = abx^2; a(bx) = (ab)x; (10-x)(10-x) = 100 + x^2 - 20x$$

Abu Kamil also wrote the *Book On Surveying and Geometry*, which was intended for government land surveyors. There, he stated the nontrivial rules for calculating areas, volumes, perimeters, and diagonals of different objects in geometry. [9]

Ibrahim ibn Sinan (908–946), a grandson of Thabit bin Qurra, was both an astronomer and a mathematician. Fuat Sezgin writes, "He was one of the most important mathematicians in the medieval Islamic world." [10] He studied geometry, and his work on calculation of the area under the graph of a parabola is especially appreciated. Going further than Archimedes, he introduced a more general method of integration. [11]

Abu Bakr ibn Muhammad ibn al-Husayn al-Karaji (953–1029), also known as al-Karkhi, is regarded as the first person to have developed algebraic operations without using geometry. One of his major works was *Al-Fakhri fi'l-jabr wa'l-muqabala* (Glorious on algebra). Historian Woepcke recognizes *Al-Fakhri* as the beginning of the theory of algebraic calculus. [12] Here, al-Karkhi introduced the monomials x , x^2 , x^3 , ... and $1/x$, $1/x^2$, $1/x^3$, ... and explained product rules among them. Moreover, he was the first to find the solutions of the equations $ax^{2n} + bx^n = c$. [13] Al-Karkhi proved the sum formula for integral cubes by using the method of proof by induction, and hence became the first to use this method. [14]

Abu'l Hasan ibn Ali al-Qalasadi (1412–1486) was an Andalusian Muslim mathematician. His main contribution was to introduce algebraic symbolism, and he used short Arabic words for his symbols. For example, he used the symbol for the sound "sh" from the Arabic word meaning "thing" to represent what we call x , the unknown. [15]

Trigonometry

Khwarizmi also contributed to trigonometry. He established accurate trigonometric tables for sine and cosine, and he was the first to introduce tangent tables. [16] In 1126, these works were translated into Latin by Adelard of Bath.

Al-Battani or Albetagnius (about 850–929) was a Muslim astronomer and mathematician. In his research on astronomy he used trigonometric methods which were a lot more advanced than the geometric methods used by Ptolemy. [17] He introduced trigonometric ratios. For example, for a right triangle with adjacent sides a and b , he gives the formula $b \sin(A) = a \sin(90^\circ - A)$, which is equivalent to $\tan A = a/b$. He was the first to introduce the cotangent function. [18]

Muhammad Abu'l Wafa (940–998), born at Buzjan in Khorasan, introduced the use of secant, cosecant and tangent functions. He gave a new method of constructing sine tables. He calculated $\sin(30^\circ)$ with an accuracy of up to eight decimal digits. He improved spherical trigonometry and proved the law of sines for general spherical triangles. [19] In particular, he developed the half/double angle formulas:

$$2 \sin^2 (x/2) = 1 - \cos x; \sin 2x = 2 \sin x \cos x$$

He was the first to introduce the notion of secant and cosecant, and hence completed the list of all six trigonometric functions. [20]

Abu Abd Allah Muhammad ibn Muadh Al-Jayyani (989–1079) was an Arab mathematician from Andalus. He was the author of *The Book of Unknown Arcs of a Sphere* which was "the first treatise on spherical trigonometry." [21] Here he mentioned formulas for right handed triangles and law of sines. He also stated the formula for the solution of a spherical triangle in terms of the polar triangle. [21] He had a strong influence on the West.

Another outstanding mathematician Nasir al-Din al-Tusi (1201–1274) wrote *Treatise On The Quadrilateral*, considered the best book on trigonometry written in medieval times, [25] later translated into French by Alexandre Carathéodory Pasha in 1891. In his book al-Tusi made enormous advances in plane and spherical trigonometry. *The Dictionary of Scientific Biography* [22] states, "This work is really the first in history on trigonometry as an independent branch of pure mathematics and the first in which all six cases for a right-angled spherical triangle are set forth." The well-known sine law is also stated in this work: $a/\sin A = b/\sin B = c/\sin C$.

Ghiyath al-Din al-Kashi (1393–1449) produces sine tables of up to eight decimal places. In 1424, he computed 2π to an accuracy of sixteen decimal digits. He wrote a very impressive book on mathematics: *Miftah al-Hussab* (Key to Arithmetic). His main purpose in this book is to provide sufficient knowledge of mathematics for those who are working on astronomy, surveying, architecture, accounting and trading. [23] He also describes how to find the fifth root of any number. [24]

Unfortunately, the contributions of Muslims often go unrecognized. Muslim scholars contributed to science in many aspects such as mathematics, astronomy, geography, philosophy, medicine, art, architecture and so on. However, today few realize that in that era Islam played an important role in all aspects of life. Europe faced losing the works of major scholars, but as a result of their translations into Arabic most of this scholarship not only survived, but was further developed. Inspired by the Qur'an and hadiths, Muslims sought knowledge for the benefit of humankind. As the Qur'an says, "Are those who know equal to those who know not?" (Zumar 39:9). We should appreciate the scholars of all eras for their contributions to science.

References

1. Tirmidhi, 'Ilm, 19.
2. B.L. van der Waerden, A History of Algebra.
3. Ibn Maja, Hadith No: 2719.
4. D. El-Dabbah, The geometrical treatise of the ninth-century Baghdad mathematicians Banu Musa (Russian), in History Methodology Natur. Sci., No. V, Math. Izdat. (Moscow, 1966), 131–139.
5. Y. Dold-Samplonius, A. T. Grigorian, B. A. Rosenfeld, Biography in Dictionary of Scientific Biography (New York 1970–1990).
6. For more, see S. Brentjes and J. P. Hogendijk, Notes on Thabit ibn Qurra and his rule for amicable numbers, Historia Math. 16 (4) (1989), 373–378.
7. R. Lorch, Abu Kamil on the pentagon and decagon, Vestigia mathematica (1993), 215–252.
8. J. Sesiano, La version latine medievale de 'l'Algebre d'Abu Kamil, in Vestigia mathematica (Amsterdam, 1993), 315–452.
9. J. Sesiano, Le Kitab al-Misaha d'Abu Kamil, Centaurus 38 (1996), 1–21.
10. F. Sezgin, History of Arabic literature (German) Vol. 5 (Leiden, 1974), 292–295.
11. <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Ibrahim.html>
12. F. Woepcke, Extrait du Fakhri, traite d'Algebre par Abou Bekr Mohammed Ben Alhacan Alkarkhi (Paris, 1853).
13. Boyer, Carl B. (1991). "The Arabic Hegemony", A History of Mathematics, Second Edition, John Wiley & Sons, Inc., 239. ISBN 0471543977.
14. Victor J. Katz (1998). History of Mathematics: An Introduction, p. 255–259. Addison-Wesley. ISBN 0321016181.
15. J. Samsó, Las ciencias de los antiguos en al-Andalus (Madrid, 1992).
16. http://en.wikipedia.org/wiki/History_of_trigonometry
17. <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Al-Battani.html>
18. <http://www.unhas.ac.id/~rhiza/saintis/battani.html>
19. <http://www.britannica.com/EBchecked/topic/2127/Abul-Wafa>
20. <http://www.bhatkallys.com/article/article.asp?aid=3442>
21. O'Connor, John J. & Robertson, Edmund F., Abu Abd Allah Muhammad ibn Muadh Al-Jayyani.
22. S. H. Nasr, Biography in Dictionary of Scientific Biography (New York 1970–1990).
23. <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Al-Kashi.html>
24. <http://www.bhatkallys.com/article/article.asp?aid=3442>
25. <http://members.tripod.com/worldupdates/newupdates10/id142.htm>
26. Heinrich Dorrie, David Antin (1965). 100 Great Problems of Elementary Mathematics: Their History and Solution, p.34–36. ISBN 0486613488.